Identification of Aircraft Longitudinal Stability and Control Derivatives Combining Global Search Natural Algorithms and a Gradient Based Algorithm

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Abstract: In this work, two optimization methods are investigated to accomplish the parameter identification of the longitudinal motion of a military aircraft within the framework of the Output Error methodology. One of these methods is based on natural algorithms, in particular, the genetic algorithms and particle swarm optimization, which combined constitute the so called Life Cycle Method. The other is the Levenberg-Marquardt optimization algorithm, which is a gradient based method. Since the methodologies differ in the way they perform the optimization, being the first based on search and so appropriate for global minima search and the second gradient based, thus, good for local minima, both are compared and used in such a way that they complement each other.

1. INTRODUCTION

The System Identification and Parameter Estimation procedures are a fundamental step for modeling and simulation, which constitutes one of the most important phases of the design and evaluation process in the aeronautical design nowadays. System Identification is a general procedure to match the observed input-output response of a dynamic system by a proper choice of an input-output model and its physical parameters. From this point of view, the aircraft system identification or inverse modeling comprises proper choice of aerodynamic models, the development of parameter estimation techniques by optimization of the mismatch error between predicted and real aircraft response and the development of proper tools for integration of the equations of motion within the system simulation and correlated activities (Jategaonkar, 2003).

The optimization methods used in the estimation problems have been traditional methods which provide local minimum searches (Nelles, 2000). In some parametric estimation cases, however, the a priori knowledge about the dynamic model can be poor and minimization with respect to the parameters of interest can result in a local minima.

Concerning the poor knowledge about the parameters of interest, therefore, the heuristic optimization methods became particularly interesting for aerodynamic parameter estimation. The genetic algorithm and the particle swarm initialization can be done in such a way that the search region contains a greater information content and the minimization procedure inclines to obtain the global minima.

It is well known that the identification by using Output Error Method associated with classical gradient-based optimization methods is a difficult task due to the existence of local minima in the design space. Moreover, such methods require an initial guess to the solution and it is not possible to assure global convergence. These aspects have motivated the authors of this paper to explore the performance of a gradient-based, namely the Levenberg-Marquardt method (LM) (Nelles, 2000), and a heuristic method, namely the LifeCycle Model (LC) (Krink and Lovberg, 2002). LM is a second order variant of the Gauss-Newton method and LC is a hybrid approach, based on two heuristic methods, namely Genetic Algorithms (GA) and Particle Swarm Optimization (PSO) combined in an evolutionary strategy. The individual performance of both algorithms are tested and then a cascade-type scheme is proposed aiming to take advantage of the global search capabilities of LC and the local search capabilities of LM.

An optimization problem is formulated so that the objective function represents the difference between the measured variables of the aircraft dynamics and its model counterpart. From a nonlinear state space model for the longitudinal motion of aircraft, a total number of ten parameters are assumed as being unknown. In this case, the parameters represent the aerodynamic stability and control derivatives of the aircraft. In order to show the optimization strategy and the approaches, experimental flight test data, performed with a military aircraft (Xavante AT-26 FAB 4516), was used.
2. PROBLEM FORMULATION

2.1 Output Error Method

In this section, the parametric identification, in particular the parameter estimation applied to a nonlinear model of the longitudinal motion of an aircraft in space state formulation is presented. The Output Error method, as shown in Fig. 1, is one of the most used estimation methods for aerodynamic parameter estimation from flight test data (Illif, 1989), (Maine and Illif, 1985). It has several desirable statistical properties, including its applicability to nonlinear dynamical systems and the proper accounting of measurements noise (Illif and Taylor, 1972).

![Fig. 1. Block diagram of the estimation procedure.](image)

The structure of the model is considered to be known, and the identification process consists in determining the parameter vector \( \Theta \), which gives the best prediction of the output signal \( y(t) \), using some sort of optimization criteria. The attainment of an estimate through optimization of a cost function based on the prediction error, requires, usually, the minimization of a nonlinear function.

Consider a dynamic system, identifiable, with model structure \( M(\Theta) \) defined and output \( y \). Suppose that \( p(y|\Theta) \) is the conditional probability gaussian distribution of the random variable \( y \) with dimension \( m \), mean \( f(\Theta) \) and covariance \( R \), with dimension \( m \times m \). \( p(y|\Theta) \) is known as the likelihood functional, and in (Goodwin and Payne, 1977) the authors attribute its name due to the fact that it is a measure of the probability of occurrence of the observation \( y \) for a given parameter \( \Theta \). Thus, the likelihood functional is:

\[
p(y|\Theta) = \frac{1}{(2\pi)^{m/2}|R|^{n/2}} e^{\frac{1}{2} \sum_{k=1}^{n} [e(k, \Theta)]^{T}[R^{-1}]^{-1}[e(k, \Theta)]}
\]

The Maximum Likelihood Estimate is defined as the value of \( \Theta \) which maximizes this functional, in such a way that the best estimate of \( \Theta \), according to the MLE criteria is:

\[
\hat{\Theta} = \arg\max \ p(y|\Theta)
\]

The maximization of \( p(y|\Theta) \) is equivalent to the minimization of \( J(\Theta) \), which is given by,

\[
J(\Theta) = \sum_{k=1}^{n} \frac{1}{2} \left[ e(k, \Theta) \right]^{T}[R^{-1}]^{-1} \left[ e(k, \Theta) \right] + \ln|R|
\]

since, in the optimization process, \( J(\Theta) \) is equivalent to \(-\ln p(y|\Theta)\), except for a constant term.

As can be seen in the general framework illustrated by Fig. 1, the Output Error has no specific requirement about the optimization method. Even though most of works in the literature reports the use of classical methods, there is no limitations in using other approaches. In this scenario, two optimization strategies are explored to solve the identification problem.

2.2 Dynamic Model

The nonlinear state equations pertaining to the longitudinal mode of aircraft motion are given by:

\[
\begin{align*}
\dot{q} &= \frac{1}{I_{y}} \left[ q\slip \bar{C}_{m}^{CG} + M_{tw} - I_{xz} (p^{2} - r^{2}) + (I_{z} - I_{x}) pr \right] \\
\dot{\theta} &= q \cos \phi - r \sin \phi \\
\dot{u} &= -qw + rv - g \sin \theta + \frac{q}{m}C_{x} + \frac{1}{m}X_{tw} \\
\dot{r} &= -pv + qu + g \cos \theta \cos \phi + \frac{2}{m}C_{z} + \frac{1}{m}Z_{tw} \\
\dot{h} &= uw \sin \theta - v \cos \theta \sin \phi - w \cos \theta \cos \phi
\end{align*}
\]

where \( C_{X}, C_{Z} \), and \( C_{CG}^{m} \) denote the aerodynamic coefficients of longitudinal force, vertical force and pitching moment. \( X_{tw} \) and \( Z_{tw} \), the thrust forces along the \( x \) and \( z \) axes and \( M_{tw} \) the pitching moment due to engine thrust about the \( y \) axis respectively.

The longitudinal and vertical force coefficients, \( C_{X} \) and \( C_{Z} \), are obtained from the lift and drag coefficients, \( C_{L} \), \( C_{D} \) and the pitching moment coefficient \( C_{m}^{CG} \) referred to the CG is given by:

\[
\begin{align*}
C_{X} &= -C_{D} \cos \alpha + C_{L} \sin \alpha \\
C_{Z} &= -C_{D} \sin \alpha - C_{L} \cos \alpha \\
C_{m}^{CG} &= C_{RP} + C_{X} \frac{x_{tueg}}{l_{p}} - C_{Z} \frac{x_{tueg}}{l_{\mu}}
\end{align*}
\]

The forces along the \( x \) and \( z \) axes and pitching moment due to engine thrust are given by:

\[
\begin{align*}
X_{tw} &= (F_{L} + F_{R}) \cos \sigma \\
Z_{tw} &= - (F_{L} + F_{R}) \sin \sigma \\
M_{tw} &= X_{tw} x_{tueg} - Z_{tw} x_{tueg}
\end{align*}
\]

where \( F_{L} \) and \( F_{R} \) are the thrusts due to left and right engines, \( \sigma \) is the thrust inclination angle with respect to the body-fixed \( x \) axis, and \( x_{tueg} \) and \( z_{tueg} \) are the \( x \) and \( z \) locations of engines from CG respectively. It is assumed that the two engines are mounted symmetrically.

The unknown aerodynamic derivatives appear in the following postulated aerodynamic model:
\[ C_L = C_{L0} + C_{La} \alpha + C_{Lq} \frac{\dot{q}_l}{V_t} + C_{Lk} \delta E \]
\[ C_D = C_{D0} + \frac{\pi e \Lambda}{\pi} C_{2} L \]
\[ C_{m} = C_{m0} + C_{ma} \alpha + C_{mq} \frac{\dot{q}_l}{V_t} + C_{m_k} \delta E \]

The drag is modelled using a drag-polar. The Oswald-factor \( e \) characterizes the increase in drag over the ideal conditions caused by non-elliptical lift distribution.

The response equations are given by:

\[ q_m = q + \Delta q \]
\[ \alpha_{aoa_m} = F_{\alpha} \tan^{-1} \left( \frac{w_{aoa}}{u_{aoa}} \right) + \Delta \alpha_{aoa} \]
\[ \theta_m = \theta + \Delta \theta \]
\[ h_m = h \]
\[ V_{tm} = \sqrt{u_{tas}^2 + u_{tas}^2 + w_{tas}^2} \]
\[ \dot{q}_m = \dot{q} \]
\[ \alpha_{x_{tas}} = \frac{a_{CG}}{x_{bscg}} - x_{bscg} (q^2 + \dot{q}^2) + y_{bscg} (p \dot{q} - \dot{r}) + z_{bscg} (p \dot{q} + \dot{r}) + z_{bscg} (p^2 + q^2) \]

where \((x_{bscg}, y_{bscg}, z_{bscg})\) are the distances of the accelerometer from the CG along x and z axes.

The previous dynamical equation has ten unknown parameters that need to be estimated, giving \( \Theta \in \mathbb{R}^{10} \),
\[ \Theta = [C_{L0}, C_{La}, C_{Lq}, C_{D0}, e, C_{m0}, C_{ma}, C_{mq}, C_{m_k}]^T \]

3. LIFE CYCLE MODEL FOR THE MINIMIZATION OF THE COST FUNCTION

3.1 The Inspiration from Nature

LifeCycle Model (LC) is a computational tool inspired in the biologic concept of life cycle. From a biology viewpoint, the term is used here to define the passage through the phases during the life of an individual. Some phases, as sexual maturity, are one-time events, others, as the mating seasons, are re-occurring. Although it does not happen in every case, the transitions between life cycle phases are started by environmental factors or by the necessity to fit to a new condition (Krink and Lovberg, 2002). The transition process promotes the maturity of an individual and contributes to the adaptation and evolution of its species.

From the implementation point of view, algorithms such as GA and PSO are heuristics search methods of proven efficiency as optimization tools. LC intends to put together the positive characteristics found in each method and creates a self-adaptive optimization approach. Each individual, as a candidate solution, decides based on its success if it would prefer to belong to a population of a GA, or to a swarm of a PSO. This means that different heuristic techniques contribute together to form a robust high performance optimization tool. The idea is that complex problems can be conveniently considered from the optimization viewpoint. As can be viewed, the less well-succeeded individuals must change their status in order to improve their fitness. In Plain English, the optimization approach does not follow a rigid scheme as proposed in (Assis and Steffen, 2003), in which various techniques are used sequentially in a cascade-type structure. In other words, it is the mechanism of self-adaptation of the optimization problem that rules the procedure. Figure 2 shows the outline of a basic LC algorithm.

Take the example of a LC with just GA and PSO as heuristics. According to Fig. 2, the algorithm is initialized with a set of particles of a PSO swarm, which can turn into GA individuals, and then, according to their performance, return to particles again, and so on. A LifeCycle individual switches its stage when there is no fitness improvement for more than a previously defined number of iterations. This is a parameter that can be adjusted according to the problem.

More detailed information about LC can be found in (Krink and Lovberg, 2002). (Rojas et al, 2004).

3.2 Genetic Algorithms

Genetic algorithms (GAs) are optimization techniques inspired by evolutionary biology concepts such as inheritance, mutation, natural selection, and recombination (or crossover). In a typical GA, a population of abstract representations (chromosomes) of candidate solutions to an optimization problem (individuals) evolves toward better solutions. The evolution starts from a population of completely random individuals and happens in generations. During the evolutionary process, each individual of the population is evaluated, reflecting its adaptation capability to the environment. Multiple individuals are stochastically selected from the current population (based on their fitness), modified (mutated or recombined) to form a new
population, which becomes current in the next iteration of the algorithm. Some of the individuals of the population are preserved while others are discarded; this process mimics the natural selection in the Darwinism.

The outline of a basic GA is as follows:

(1) Define the GA parameters (population size, selection method, crossover method, mutation rate, etc.).
(2) Create an initial population, randomly distributed throughout the design space (other distributions can be performed).
(3) Evaluate the objective function and take it as a fitness measure of each individual.
(4) Select mates to the crossover; this mimics the natural selection.
(5) Reproduce and replace the worst individuals in the population by the offspring.
(6) Mutate, to avoid premature convergence (other parts of the design space are explored).
(7) Go to step 3 and repeat until the stop criterion is achieved.

Although the initial proposed GA was dedicated to discrete variables, nowadays, improvements are available to deal with discrete and continuous variables, see (Haupt and Haupt, 2004), (Michalewicz and Fogel, 2000) for more details.

3.3 Particle Swarm Optimization

Particle swarm optimization (PSO) is a technique inspired on the swarm intelligence, the study of collective behavior details..

3.3.1 Overview

Particle swarm optimization (PSO) is inspired on the swarm intelligence, the study of collective behavior of a swarm of birds when looking for food or nest. This is achieved by modeling the flying using a velocity vector. The velocity vector considers a contribution of the current velocity and other two portions referred to the knowledge of the particle itself and of the swarm, respectively, about the search space. In such a way, the velocity vector is used to update the position of each particle in the swarm.

It can be seen below an outline of a basic PSO algorithm.

(1) Define the PSO parameters (swarm size, inertia value, "trust" parameters, etc.).
(2) Create an initial swarm.
(3) Update the velocity vector for each particle.
(4) Update the position for each particle.
(5) Go to step 3 and repeat until the stop criterion is achieved.

For learn more about PSO, see (Kennedy and Eberhart, 1995) and (Venter and Sobieszczanski-Sobieski, 2002).

4. LEVENBERG-MARQUARDT OPTIMIZATION METHOD

The Levenberg-Marquardt method is a second order technique based on the Gauss-Newton method. The update of the parameters is given by the following equation

$$\Theta_{i+1} = \Theta_i - \left[ \nabla^2_\Theta J(\Theta_i) \right]^{-1} \nabla_\Theta J(\Theta_i)$$

(10)

The complexity in the calculation of the Hessian matrix, $\nabla^2_\Theta J(\Theta_i)$ in (10), is avoided through the Gauss-Newton method, which uses the approximation,

$$\nabla^2_\Theta J(\Theta) \approx \sum_{k=1}^{n} [\nabla_\Theta \hat{y}_k(\Theta)]^T \left[ \hat{R} \right]^{-1} [\nabla_\Theta \hat{y}_k(\Theta)]$$

(11)

where the terms involving the second derivatives are discarded. The gradient of the estimated output, $\nabla_\Theta \hat{y}_k(\Theta)$, is called Sensitivity Function.

The Levenberg-Marquardt algorithm is an extension of the Gauss-Newton algorithm (Nelles, 2000). The main idea is to modify the Hessian to

$$\nabla^2_\Theta J(\Theta) \approx \sum_{k=1}^{n} [\nabla_\Theta \hat{y}_k(\Theta)]^T \left[ \hat{R} \right]^{-1} [\nabla_\Theta \hat{y}_k(\Theta)] + \lambda I$$

(12)

where the inversion of the matrix 12 is not performed explicitly, but solving by Singular Value Decomposition (SVD) the following expression

$$\left[ \nabla^2_\Theta J(\Theta) + \lambda I \right] \Delta \hat{\Theta} = \nabla_\Theta J(\Theta)$$

(13)

The addition of the term $\lambda I$ in (12) solves the problem of ill-conditioning of the matrix. The Levenberg-Marquardt algorithm can be interpreted in the following manner: for small values of $\lambda$ it behaves like the Gauss-Newton algorithm, while for high values of $\lambda$ it behaves like the steepest gradient algorithm.

5. RESULTS

The identification of longitudinal stability derivatives from flight test data were performed using a set of experimental flight data performed with a military aircraft, namely the Xavante (AT-26 FAB 4516).

Table 1 gives the setup for LifeCycle used in experimental tests.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Search space</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0$</td>
<td>-0.01 0.01</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>-0.20 0.20</td>
</tr>
<tr>
<td>$w_0$</td>
<td>70 110</td>
</tr>
<tr>
<td>$w_1$</td>
<td>8 12</td>
</tr>
<tr>
<td>$h_0$</td>
<td>900 9400</td>
</tr>
<tr>
<td>$C_{La}$</td>
<td>0 0.30</td>
</tr>
<tr>
<td>$C_{L_{13}}$</td>
<td>4 8</td>
</tr>
<tr>
<td>$C_{L_{14}}$</td>
<td>-20 -1</td>
</tr>
<tr>
<td>$C_{L_{k3}}$</td>
<td>-2 -0.1</td>
</tr>
<tr>
<td>$C_{D_{2k}}$</td>
<td>0 0.02</td>
</tr>
<tr>
<td>$C_{C_{m_{12}}}$</td>
<td>0 0.1</td>
</tr>
<tr>
<td>$C_{m_{12}}$</td>
<td>-0.5 -0.1</td>
</tr>
<tr>
<td>$C_{C_{m_{12}}}$</td>
<td>-10 -2</td>
</tr>
<tr>
<td>$C_{m_{d_2}}$</td>
<td>-1.0 -0.1</td>
</tr>
</tbody>
</table>

| Population size | 500 |
| Stage interval  | 10  |

For the first strategy, the behavior of the LC along the iterations and the transitions due to its self-adaptation skills can be observed in Fig. 3. In this figure it’s possible to infer which heuristics is conducting the optimization process at a given iteration.
Fig. 3. LC evolution and performance.

Table 2. Comparison of parameter estimation.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>LC</th>
<th>LC-LM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>-0.0013</td>
<td>-0.00517 (20.87 %)</td>
</tr>
<tr>
<td>$b_0$</td>
<td>0.1488</td>
<td>0.1467 (0.47 %)</td>
</tr>
<tr>
<td>$a_1$</td>
<td>92.36</td>
<td>77.87 (0.048 %)</td>
</tr>
<tr>
<td>$b_1$</td>
<td>11.47</td>
<td>11.9 (0.4142 %)</td>
</tr>
<tr>
<td>$a_2$</td>
<td>9309</td>
<td>9352 (0.0032 %)</td>
</tr>
<tr>
<td>$C_{LC}$</td>
<td>0.0089</td>
<td>0.2278 (3.3129 %)</td>
</tr>
<tr>
<td>$C_{L_{L_{LC}}}$</td>
<td>6.7892</td>
<td>6.7632 (0.62 %)</td>
</tr>
<tr>
<td>$C_{L_{L_{LC}}}$</td>
<td>-3.4611</td>
<td>-51.2799 (8.14 %)</td>
</tr>
<tr>
<td>$C_{L_{L_{LC}}}$</td>
<td>-0.5154</td>
<td>-1.4083 (7.46 %)</td>
</tr>
<tr>
<td>$C_{D_{N}}$</td>
<td>0.001297</td>
<td>0.00256 (6.88 %)</td>
</tr>
<tr>
<td>$C_{m_0}$</td>
<td>0.0484</td>
<td>0.060 (0.51 %)</td>
</tr>
<tr>
<td>$C_{m_0}$</td>
<td>-0.4319</td>
<td>-0.3978 (-0.4198 %)</td>
</tr>
<tr>
<td>$C_{m_1}$</td>
<td>-3.8981</td>
<td>-0.3666 (46.27 %)</td>
</tr>
<tr>
<td>$C_{m_2}$</td>
<td>-0.5643</td>
<td>-0.6627 (0.6918 %)</td>
</tr>
</tbody>
</table>

Cost function | 1.5496e-10 | 1.2874e-16 |
Iterations | 90 | 92 |
Time [s] | 35.47 | 768 |

Table 2 reports a set of identification results obtained following the two different optimization strategies. For the case in which LM were used, the values in parentheses refers to an estimate of the relative standard deviation, calculated from the Fischer information matrix. Figures with the comparison of measured and estimated values demonstrates the prediction capacity of the LC-LM approach.

6. CONCLUSION

This paper presented an identification procedure to determine the longitudinal stability and control derivatives of a military aircraft from a nonlinear model. The tested inverse problem solvers are based on Life Cycle and Levenberg-Marquardt methods. In the present contribution, GA and PSO represented the phases of the LC algorithm. A cascade-type scheme is proposed using both optimization algorithms, aiming to take advantage of the global search capabilities of LC and the local search capabilities of LM. Finally, the experimental investigation illustrated the possibility of using the present technique in real world environment. The results are very encouraging in the sense that with little knowledge of the aircraft aerodynamic derivatives, it is possible to start up the local search gradient based algorithm, which is very sensitive to initial values of parameters.

7. ACKNOWLEDGEMENTS

The authors are thankful to the Brazilian Research Agency, CNPq, and Fundação de Amparo à Pesquisa do Estado de São Paulo, FAPESP, for the grant of Ph.D. and research scholarships.

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